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PROCEEDINGS BOOK

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CONTENT

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DETERMINATION OF EIGENFORMS OF LIQUID OSCILLATIONS IN TANKS WITH THE WINKLER ELASTIC BASE

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Basic functions ${}_{2l}$ and their corresponding natural frequencies ${}_{l}$ determined by solving the boundary value problem (1).

$$\nabla^2{}_{2k} = 0, \left.\frac{\partial_{2k}}{\partial \mathbf{n}}\right|_{S_1} = 0, \left.\frac{\partial_{2k}}{\partial \mathbf{n}}\right|_{S_{bot}} = 0, \left.\frac{\partial_{2k}}{\partial \mathbf{n}}\right|_{S_0} = \frac{\partial}{\partial t}, \quad \frac{\partial_{2k}}{\partial t} + g\zeta = 0 \qquad (1)$$

Note due to the circular symmetry of the structure under consideration, the equations could be represented as products of trigonometric functions cos$m$ on functions depending on r and z. Here m is wave number, or the number of nodal diameters. For a rigid cylindrical shell of radius R with fill level H , Fig. 1, own functions ${}_{2k}$ and natural oscillation frequencies χk there have been got by the formulas [1-3]

$$\frac{{}_k^2}{g} = \frac{{}_k}{R}\tanh\left({}_k\frac{H}{R}\right), \; {}_{2k} = \mathrm{J}_m\left(\frac{{}_k}{R}r\right)\frac{\cosh\left(\frac{{}_k}{R}(z+H)\right)}{\cosh\left(\frac{{}_k}{R}(H)\right)}\cos m, \qquad (2)$$

where J_m are Bessel functions of the first kind, ${}_k$ are roots of the equation $\mathrm{J}_m(x) = 0$.
Furthermore, there have been considered the boundary value problems for each wave number separately. At the same time, it has been had the following orthogonality relations

$$\int_0^R r\mathrm{J}_m\left(\frac{{}_k}{R}r\right)\mathrm{J}_m\left(\frac{{}_l}{R}r\right)dr = 0, kl; \quad \int_0^R r\mathrm{J}_m^2\left(\frac{{}_k}{R}r\right)dr = \frac{R^2}{2}\left[\left(1\frac{m^2}{{}_k^2}\right)\mathrm{J}_m^2({}_k)\right].$$

Values of the equations roots $J_m(x) = 0$ have been presented in Table 1 with different m.

Table 1. Characteristic numbers

k	m=0	m=1	m=2	m=3
1	3.831705970	1.841183781	3.054236928	4.201188941
2	7.015586670	5.331442775	6.706133195	8.015236600
3	10.17346814	8.536316365	9.969467825	11.34592431
4	13.32369194	11.70600490	13.17037086	14.58584829
5	16.47063005	14.86358863	16.34752232	17.78874787
6	19.61585851	18.01552786	19.51291278	20.97247694

From the data in the table, there could be seen the lowest frequencies will correspond to the first harmonic, m=1.

Determination of natural forms of the bottom vibrations

Basic functions w_l and their corresponding natural frequencies ${}_l$ determined by solving such a spectral problem

$$D\Delta\Delta w_k + (K\,\rho_p h_k^2) w_k = 0, \qquad (3)$$

$$w\big|_{r=R} = 0, \quad \frac{dw}{dr}\bigg|_{r=R} = 0. \qquad (4)$$

Since equation (28) allows solutions in the form

$$w_{km}(r,) = F_k(r)\cos m,$$

then in view of the equations (3), (4) there have been concluded the equation (3) allows reduction by $\cos m$.

First, it has been considered the case of axially symmetric oscillations, that is, let's assume that m=0. Let's introduce the following notation

$${}^4 = \frac{{}^2\rho_p h}{D}\frac{K}{D}.$$

Equation (3) takes the form $\left(\Delta - \alpha^2\right)\left(\Delta + \alpha^2\right)F = 0$ and could be depicted as a system

$$\frac{d^2F}{dr^2} + \frac{1}{r}\frac{dF}{dr} - \alpha^2 F = 0, \qquad (5)$$

$$\frac{d^2F}{dr^2} + \frac{1}{r}\frac{dF}{dr} + \alpha^2 F = 0. \qquad (6)$$

The solutions of equation (5) are Bessel functions of the first and second kind of zero order $J_0(\alpha t)$ and $Y_0(\alpha r)$, and the solutions of equation (6) are modified Bessel functions of the first and second kind of zero order $\mathrm{I}_0(\alpha r)$ and $K_0(\alpha r)$. Thus, the general solution of equation (3) has the form

$$F(r) = a\mathrm{J}_0(\alpha \mathrm{r}) + bY_0(\alpha r) + c\mathrm{I}_0(\alpha \mathrm{r}) + d\mathrm{K}_0(\alpha r),$$

where a, b, c, d are constants.

Since at $r \to 0$ functions $Y_0(\alpha r)$ and $K_0(\alpha r)$ grow endlessly, it has been assumed that $b = 0,\ d = 0$, to avoid non-physical movements. Then to avoid non-physical movements a, c it has been used the boundary conditions for fixing the plate along the contour. In the case of rigid fixation, there have been obtained the following boundary conditions

$$F\big|_{r=R} = 0, \quad \frac{dF}{dr}\bigg|_{r=R}.$$

Thus

$$\begin{cases} a\mathrm{J}_0(\alpha R) + c\mathrm{I}_0(\alpha R) = 0 \\ a\mathrm{J}_1(\alpha R) + c\mathrm{I}_1(\alpha R) = 0 \end{cases} \qquad (7)$$

In order for the system (7) to have a nonzero solution, it is necessary that the determinant of this system is equal to zero. Therefore, there have been got the characteristic equation for finding the unknown quantity α

$$\begin{vmatrix} \mathrm{J}_0(\alpha R) & \mathrm{I}_0(\alpha R) \\ \mathrm{J}_1(\alpha R) & \mathrm{I}_1(\alpha R) \end{vmatrix} = \mathrm{J}_0(\alpha R)\mathrm{I}_1(\alpha R) - \mathrm{I}_0(\alpha R)\mathrm{J}_1(\alpha R) = 0. \qquad (8)$$

It has been marked $\lambda = \alpha R$. Table 2 shows the values of the first 6 roots of equation (8) at m=0.

Table 2. Values of the roots of the characteristic equation (8) and constants c_k

k	λ_k	c_k
1	3.196220616	0.1018870979
2	6.306437050	0.0506907858
3	9.439499140	0.0337792448
4	12.57713064	0.0253319976
5	15.71643853	0.0202649244
6	18.85654552	0.0168871927

The ratio between constants a and c in the equation for w for every α_k have been got from equality

$$a_k \mathrm{J}_0\left(\alpha_k R\right) + c_k \mathrm{I}_0\left(\alpha_k R\right) = 0 \Rightarrow c_k = -\, a_k \frac{\mathrm{J}_0\left(\alpha_k R\right)}{\mathrm{I}_0\left(\alpha_k R\right)}.$$

Thus, the dependences of the forms of natural oscillations of a round plate on r have been obtained in the form

$$w_k(r) = \mathrm{J}_0\left(\alpha_k r\right) - \frac{\mathrm{J}_0\left(\alpha_k R\right)}{\mathrm{I}_0\left(\alpha_k R\right)} \mathrm{I}_0\left(\alpha_k r\right). \tag{9}$$

Figure 1 shows the functions defined by formula (9) at $R = 1$ for different k depending on r at m=0.

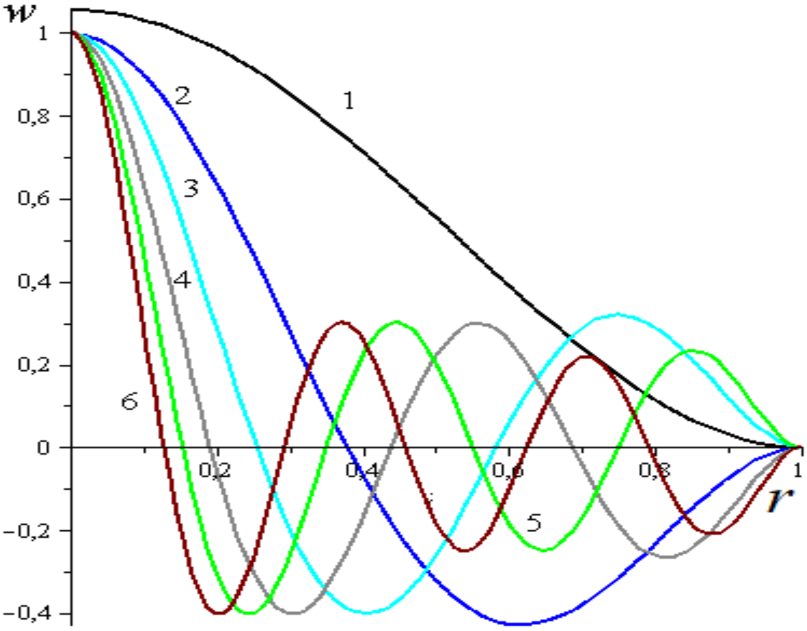

Figure 1 Dependence of the forms of oscillations on the radial coordinate

The numbers 1-6 here indicate the forms corresponding to the values $\lambda_k = \alpha_k$, which have been given in table 1. By checking the orthogonality of the forms of natural oscillations, w_m(r) it has been established, that $\left(w_k(r), w_l(r)\right) = c_k \delta_{kl}$, where are the constants values c_k have been listed in Table 1. Using the obtained orthogonality conditions and expressions (2) for the functions ${}_{2k}(r)$ and (9) for functions $w_k(r)$ at $z = -H$ it has been got the value of the scalar products $({}_{2k}, w_l)$ in the first of the equations [4-8].

Table 3. Value of scalar products $(w_l, {}_{2k})$

$k\,/l$	1	2	3	4	5	6
1	0.062175	-0.002222	0.000404	-0.000119	0.0000459	-0.000021
2	0.030527	0.0369651	-0.002831	0.007554	-0.000282	0.0001273
3	-0.015242	0.0159007	0.026325	-0.002707	0.0008747	-0.000375
4	0.0098645	-0.008068	0.010601	0.0204379	-0.002457	0.0008892
5	-0.007085	0.0054783	-0.005309	0.0079168	0.0167006	-0.002212
6	0.0054112	-0.004104	0.003659	-0.003901	0.00630544	0.0141181

From the data in Table 3, it could be concluded the largest contribution is given by the scalar products at k,l=1,2,3.

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