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MODELING THE MOVEMENT OF HETEROGENOUS FLOWS OF PEOPLE AS A GEOMETRIC DESIGN PROBLEM

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Abstract. *The problem of modeling the movement of heterogenous flows of people is shown to be one of the topical applied problems of the geometric design class. The article proposes a mathematical model, methods, and algorithms for modeling the movement of heterogenous flows of people based on the local optimization methods for the movement of geometric objects taking into account changes in their spatial shape and metric characteristics. These algorithms are based on an analytical description of the conditions for non-intersection of objects taking into account their continuous transitions and rotations.*

Keywords: *configuration space, generalized variables, placement, modeling of the movement of human flows, mathematical model.*

INTRODUCTION. PROBLEM STATEMENT

The problems of placement, coverage, and partition, which serve as models for many relevant practical problems, belong to the class of geometric design problems that are reduced to the optimized mapping of the input geometric information into a certain abstract set of the corresponding structure by performing the given set of constraints.

One of such problems is the problem of heterogenous people flow movement that allows us to promptly estimate the time that people were moving and to make a decision on safe passages.

Even today, models and methods of people and vehicle flow movement stay relevant for sectors whose formalization is not enough to implement available models and methods, which is stipulated by the need to take the features of the subject area into account. This also determines the need for constructing new mathematical models, formulating problem statements, and developing new efficient methods and algorithms for their solution.

In this article, we constructed a mathematical model for one separate problem of this class based on the analysis of current problems of modeling human flow movement whose features determine its solving approaches and allow us to use the known methods while developing new methods and algorithms.

ANALYZING LATEST RESEARCH AND PUBLICATIONS

The individualistic human movement flow models represent human flow as a set of individuals in the form of geometric shapes taking into account their form and metric characteristics. The change in the comfortability of human flow movement depends on both the forms of objects and on the feasible distance between individuals [1, 2].

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The change in the spatial form of an object under forceful actions can be formalized by representing the human body projection in the form of complex objects forming a combination of ellipses where the main one is performing a continuous rotation within the maneuverability angle, while the auxiliary ones are making continuous rotations around the "adhesion" points of auxiliary ellipses with the main one taking into account anthropological restrictions [2].

To characterize the mutual positioning of individuals in a human flow, the conditions of pairwise non-intersection of geometric objects [2, 3] are used. Special coefficients [2] are used to calculate velocity, maneuverability, and deviation from the main movement direction.

Based on the results of analyzing program packages of human flow movement modeling, there are no individualistic human flow movement models that would be adequate to the real flow [4]. The relevance of such models is stipulated by the need for studying the movement of people with restricted mobility in a mixed flow in a significantly broad nomenclature of public buildings of different functional fire hazard classes.

CONFIGURING SPACE AND GENERAL CHANGES IN THE PROBLEM OF COMPLEX GEOMETRIC SHAPE PLACEMENT. PLACEMENT CONFIGURATION

Configuration space of geometric objects is based on formalizing the geometric information concept. Geometric information $G = (\{s\}, \{\mu\}, \{u\})$ on the ellipse E includes data about its spatial form $\{s\}$, metric characteristics $\{a, b\}$ (a and b are ellipse semi-axes), and placement parameters $\{u\}$. Let us represent the spatial form $\{s\}$ of the geometric object by its boundary equation in the form $f(\mu, u) = 0$ where $u = (x, y, \theta)$ and $\mu = (a, b)$ are constants characterizing its mathematical properties, which we call parameters of spatial form s of the object.

Let us connect a coordinate eigensystem to an object E with the system origin being object poles. In the case of affine transformations, object movement changes the placement of its coordinate eigensystem in relation to the fixed coordinate system of the space R^2. To characterize this state, let us determine the placement parameters $u = (x, y, \theta)$ where $\upsilon = (x, y)$ is the translation vector in relation to the fixed coordinate system and θ is the rotation angle.

Let us construct the configuration space ΞE of the object E with generalized variables, namely, metric parameters $\mu = (a, b)$ and placement parameters $u = (x, y, \theta)$. Then, each point $G = (\mu, u) = (a, b, x, y, \theta)$ of the configuration increment ΞE determines the geometric object $E(G) \subset R^2$.

Let us perform parametrization on the subset $S_i = \{E_c^i, E_l^i, E_r^i\}$, $i \in I_n = \{1, 2, \ldots, n\}$. Using theoretical and set operations, let us construct the following complex object:

$$H_i = E_c^i \cup E_l^i \cup E_r^i. \tag{1}$$

Let us call E_c^i, E_l^i, and E_r^i base objects.

Let the object E_c^i have the form s_c^i, as well as metric parameters $\mu_c^i = (a_c^i, b_c^i)$ and placement parameters $u_c^i = (x_c^i, y_c^i, \theta_c^i)$. The objects E_l^i and E_r^i have the forms s_l^i and s_r^i, as well as metric parameters $\mu_l^i = (a_l^i, b_l^i)$ and $\mu_r^i = (a_r^i, b_r^i)$ and placement parameters $u_l^i = (x_l^i, y_l^i, \theta_l^i)$ and $u_r^i = (x_r^i, y_r^i, \theta_r^i)$, respectively.

Let $\Sigma = \{H_1, H_2, \ldots, H_i, \ldots, H_n\}$ be the output set of geometric objects with generalized variables $G_{ii}^i = (\mu_{ii}^i, u_{ii}^i)$, $ii = c, l, r$; $i = 1, 2, \ldots, n$, and let $\{s_c, s_l, s_r\}$ be a set of their feasible spatial forms (in this case, ellipses). Each point $G_{ii}^i \in \Xi(H_i)$ corresponds to a parametrized geometric object $H_i(G_{ii}^i) \subset R^2$. The configuration space has the form $\Xi\Sigma = \Xi H_1 \times \Xi H_2 \times \ldots \times \Xi H_n$ with the generalized variables $G = (G_c^i, G_l^i, G_r^i)$, $i = 1, 2, \ldots, n$.

Using theoretical and set operations, let us form a complex geometric object $S_p = P(H_1, H_2, \ldots, H_i, \ldots, H_n)$ ($S_p = H_1 \cup H_2 \cup \ldots \cup H_i \cup \ldots \cup H_n$). The operator $P: \Sigma \to S_p$ determines the structure of the complex object. Then, the complex object S_p in the configuration space $\Xi\Sigma$ corresponds to the parametrized geometric object $S^p(H_1(G_c^1, G_l^1, G_r^1), \ldots, H_n(G_c^n, G_l^n, G_r^n)) = P(H_1(G_c^1, G_l^1, G_r^1), \ldots, H_n(G_c^n, G_l^n, G_r^n))$.

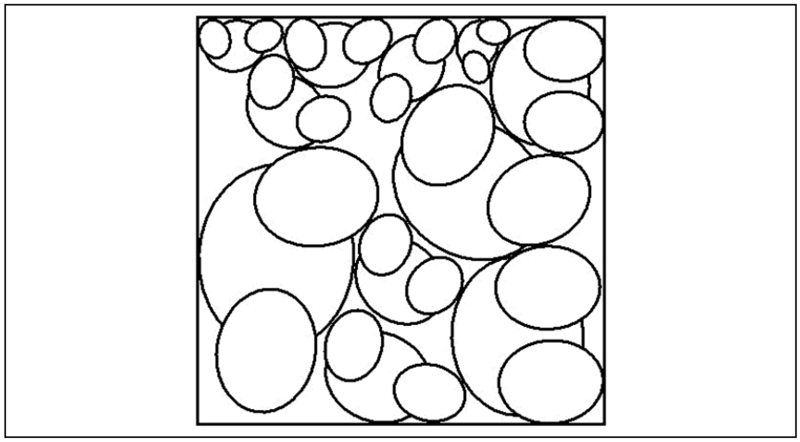

Fig. 1. The placement configuration of three-component objects taking into account continuous translations of the main (large) ellipses under the condition of continuous spinning of the main ellipse and the auxiliary ellipses that make up the complex object.

Definition 1 [5, 6]. The mapping $w: \Sigma \to \Xi\Sigma$ of the geometric set $\Sigma = \{H_1, H_2, \ldots, H_i, \ldots, H_n\}$ of objects into the configuration space $\Xi\Sigma$, which satisfies problem constraints, determines the spatial configuration of the geometric objects H_i, $i = 1, 2, \ldots, n$.

Let us denote by ΞS_0 the configuration space of the domain (object) S_0 with the generalized variables $G_0 = (\mu^0, u^0)$, and let $v^0(0, 0)$ be the origin of its fixed coordinate eigensystem and $\mu^0 = (L, W)$. Let $\Sigma^0 = \Sigma \bigcup S_0$ as well, and let us construct the configuration space $\Xi\Sigma^0 = \Xi S_0 \times \Xi H_1 \times \Xi H_2 \times \ldots \times \Xi H_n$ similar to the above.

Let us introduce the concept of spatial placement configuration. To form a system Λ of constraints, let us set binary relations [5] on the object set from the domain Σ^0 as follows:

(a) the non-intersection of $\{*\}$, i.e., $H_i(G_{ii}^i) * H_j(G_{ii}^j)$ if $\operatorname{int} H_i(G_{ii}^i) \bigcap \operatorname{int} H_j(G_{ii}^j) = \varnothing$, $i < j+1 \in I_{n-1}$, $ii = c, l, r$;

(b) the inclusion of $\{\circ\}$, i.e., $H_i(G_{ii}^i) \circ S_0(G_0)$ if $\operatorname{int} H_i(G_{ii}^i) \subset S_0(G_0)$ $\forall i = 1, 2, \ldots, n$.

Definition 2. The mapping $w: \Sigma^0 \to \Xi\Sigma^0$ determines the placement configuration if $H_i(G_{ii}^i) * H_j(G_{ii}^j)$, $H_i(G_{ii}^i) \circ S_0(G_0)$ $\forall i, j \in I_n$, $ii = c, l, r$.

An example of the placement configuration of three-component objects is presented in Fig. 1.

PROBLEM OF MODELING HETEROGENOUS HUMAN FLOW MOVEMENT AS A GEOMETRIC PROJECTION

Let us consider the problem of modeling human movement along the bodily boundary [7], which is on the horizontal plane, and let us denote it by the domain S_0. Let us partition this domain into subdomains (with the numbers $1, 2, \ldots, m$, respectively) that are constrained by straight lines (called partitions), for which conditions $A_i \in \operatorname{Fr} S_0$, $i = 1, 2, \ldots, m-1$ (Fig. 2) are fulfilled. To determine the main movement direction, let us denote the mth domain by S_{0m}. Here, the partitioner performs the translation for domains with direct movement in such a manner that the analyzed point belongs to it. Let us determine the movement from the analyzed point for the domains in the form of a vector connecting the given point with the point on the corresponding partitioner (taking into account the homothety coefficient). For each current point with the coordinates $g^i(x^i, y^i)$ (placement coordinates of the ith person), we determine the velocity vector $\bar{v}_i(x^i, y^i)$. The vector depends on the local flow density that should not exceed the permissible values.

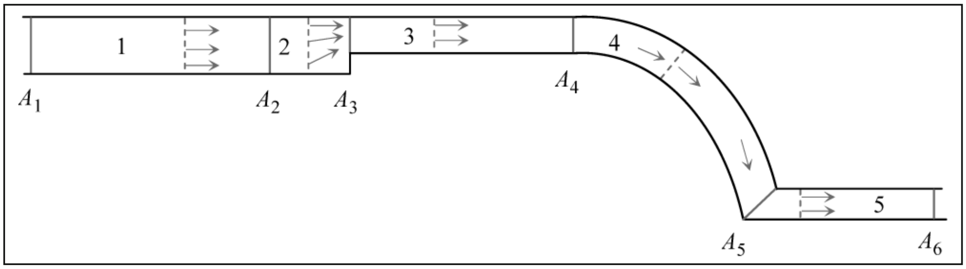

Fig. 2. Image of the movement path.

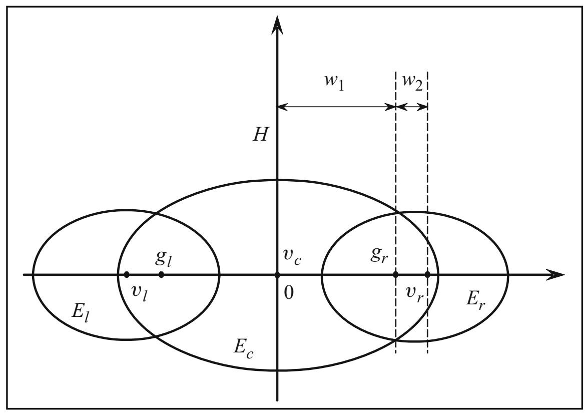

Fig. 3. A three-component model of human body mapping onto the horizontal plane.

Without restricting the generality of ideas, we assume that every individual can be represented by a three-component model by taking the conditions of combining model components into a single complex object (at the "adhesion" points) into account, as well as constraining the rotation angle of the components around these points [2]. Let us denote the horizontal projection of the human body by the combination of three ellipses, namely, the main one, E_c, and the two auxiliary ones, E_l and E_r (Fig. 3). The ellipses E_c and E_r have the adhesion point g_r, while ellipses E_c and E_l have the adhesion point g_l. The points g_r and g_l belong to the big semi-axis of the ellipse E_c, and they are placed symmetrically to its small semi-axis. The placement of the points g_r and g_l on the plane is determined only by the placement parameters of the ellipse E_c. The ellipses E_l and E_r can only rotate at the angles within the given range $(-\alpha_1, +\alpha_2)$ (in relation to the rotation angle of the ellipse E_c) in relation to these points.

Let us determine the main direction for each individual who found themselves in the movement area at ефср kth step (with the given time interval of, for example $\Delta t_k = 1\,\text{sec}$). Then, individual characteristics (such as velocity, direction, maneuverability, etc.) are slightly changed. The rotation angle of the complex object is the angle between the perpendicular to the big semi-axis of the main ellipse and the main movement direction vector.

This article formulates the mathematical model of the subproblem on the kth iteration as finding the maximum of the mutual movement of n people, which are in the domain S_0, in a movement direction in the time Δt_k taking into account the constraint on their non-intersection conditions, the conditions of people placement on a plot while adhering to the given minimal feasible distances that were determined by technological restrictions (movement comfortability, movement maneuverability constraints, etc.).

Let us show that the stated problem belongs to the geometric projection problem class [7] and is reduced to the mapping of an output set of elements of arbitrary nature into an abstract set of the corresponding structure under the given constraints [7]. This mapping is called configuration, and it is performed in the configuration space [5, 6]. Let us introduce these concepts for the problem under study.

Note that the following additional constraints [2] were put onto object (1):

(i) conditions of adhesion of three ellipses in the form of a single complex object H_i, namely,

$$x_c^i + w_1^i \cos\theta_c^i = x_r^i - w_2^i \cos\theta_r^i, \tag{2}$$

$$y_c^i + w_1^i \sin\theta_c^i = y_r^i - w_2^i \sin\theta_r^i, \tag{3}$$

$$x_c^i + w_1^i \cos\theta_c^i = x_l^i - w_2^i \cos\theta_l^i, \tag{4}$$

$$y_c^i + w_1^i \sin\theta_c^i = y_l^i - w_2^i \sin\theta_l^i; \tag{5}$$

(ii) constraints on relationships between the rotation angles that arise from the physical restrictions on the mutual placement of human body parts (for example, shoulders), namely,

$$\theta_c^i - \alpha_1^i \le \theta_l^i \le \theta_c^i + \alpha_2^i, \tag{6}$$

$$\theta_c^i - \alpha_1^i \le \theta_r^i \le \theta_c^i + \alpha_2^i. \tag{7}$$

Constraints (2)–(7) are obtained from the anthropological human data. Here, parameters $\upsilon_l^i(x_l^i, y_l^i)$ and $\upsilon_r^i(x_r^i, y_r^i)$ are determined by the placement parameters of the ellipse E_c^i (2)–(5), while the parameters of the spatial form of the objects $H_i = E_c^i \cup E_l^i \cup E_r^i, i = 1, 2, \ldots, n$, determine parameters $\mu_l^i = \theta_l^i$, $\mu_r^i = \theta_r^i$, for which constraints (6) and (7) are fulfilled.

Let us analyze the features of mapping in the human movement modeling problem, namely, its generalized parameters.

Note that additional technical restrictions are placed upon the constraints on non-intersection and object placement in the human movement modeling problems. These are constraints on the relative step in time Δt^i of the movement of the ith person, on the maneuverability z_c^i (i.e., the possibility of every human to deviate from the main movement direction), on the permissible local flow density D_{per}, and on movement comfortability (that is set either by different minimally permissible distances r_{ij}, $i < j+1 \in I_{n-1}$, between themselves and r_i, $i \in I_n = 1, 2, \ldots, n$, i.e., between the people and the area boundary or different spatial shapes by the change in the parameters μ_l^i and μ_r^i in the case when complex object (2)–(7) is created). Taking these minimally permissible distances into account is possible through the analytical description of the conditions of non-intersection and of object placement, namely, the Φ-functions [3].

The human flow movement problem belongs to continuous problems. To model movement, time discretization $t = t_0 + k\Delta t_k$, $k = 1, 2, \ldots$, is introduced that allows us to reduce the solution to the initial problem to the solution to the continuous problem sequence at each discrete time interval Δt_k where t_0 is the initial time moment.

Thus, the generalized variables of the problem of placing objects $H_1, H_2, \ldots, H_i, \ldots, H_n$ are variables $(x_c^1, y_c^1, \theta_c^1, \theta_l^1, \theta_r^1, \ldots, x_c^i, y_c^i, \theta_c^i, \theta_l^i, \theta_r^i, \ldots, x_c^n, y_c^n, \theta_c^n, \theta_l^n, \theta_r^n)$, and for the continuous problem of modeling movements of individuals who correspond to objects $H_1, H_2, \ldots, H_i, \ldots, H_n$, at the time interval Δt_k, the generalized variable vector has the following form:

$$G = (\Delta t^1, z_c^1, x_c^1, y_c^1, \theta_c^1, \theta_l^1, \theta_r^1, \ldots, \Delta t^i, z_c^i, x_c^i, y_c^i, \theta_c^i, \theta_l^i, \theta_r^i, \ldots, \Delta t^n, z_c^n, x_c^n, y_c^n, \theta_c^n, \theta_l^n, \theta_r^n).$$

Then, the human movement modeling problem at each step Δt_k can be stated as follows.

We have to find the vector of parameters

$$G^* = ((\Delta t^1, z_c^1, x_c^1, y_c^1, \theta_c^1, \theta_l^1, \theta_r^1)^*, \ldots, (\Delta t^i, z_c^i, x_c^i, y_c^i, \theta_c^i, \theta_l^i, \theta_r^i)^*, \ldots, (\Delta t^n, z_c^n, x_c^n, y_c^n, \theta_c^n, \theta_l^n, \theta_r^n)^*) \tag{8}$$

satisfying the following constraints:

— the conditions of non-intersection of objects $H_i(x_c^i, y_c^i, \theta_c^i)$, $H_j(x_c^j, y_c^j, \theta_c^j)$, $i < j+1 \in I_{n-1}$:

$$\Phi^{H_i H_j}(x_c^i, y_c^i, \theta_c^i, x_c^j, y_c^j, \theta_c^j) - r_{ij} \ge 0; \tag{9}$$

— the conditions of placing objects $H_i(x_c^i, y_c^i, \theta_c^i)$ in the domain S_0:

$$\Phi^{H_i S_0}(x_c^i, y_c^i, \theta_c^i) - r_i \ge 0,\ \ i \in I_n = 1, 2, \ldots, n; \tag{10}$$

— technical requirements $T_i(x_c^i, y_c^i, \theta_c^i) \ge 0$:

$$T_i(x_c^i, y_c^i, \theta_c^i) \ge 0: \begin{cases} 0 \le \Delta t^i \le 1, \\ \theta_c^i - \alpha_c^i \le z_c^i \le \theta_c^i + \alpha_c^i, \\ D_i(x_c^i + \upsilon_{i,x} \Delta t_k^i \Delta t_k;\ y_c^i + \upsilon_{i,y} \Delta t_k^i \Delta t_k) \le D_{\text{per}} \end{cases} \tag{11}$$

to reach the maximum cumulative human movement during the kth time interval, i.e.,

$$G^* = \arg \max_{G \in W_k \subset (R^{7n})} F(G), \tag{12}$$

$$F(G^*) \to \max_{G \in W_k \subset (R^{7n})},$$

$F(G) = \Delta t \sum_{i=1}^{n} \Delta t^i (G_i) |\vec{\upsilon}_i|$, $\Delta t^i = \dfrac{\Delta t_k^i}{\Delta t_k}$, where Δt_k^i and Δt^i are the real and relative movement time of the ith person during the time interval Δt_k, respectively.

Conditional optimization problem (12) with constraints (2)–(7) and (9)–(11) is an NP-complex nonlinear programming problem. The feasible solution domain W_k has a complex structure as it is a disconnected set where each connectivity component is multiconnected and the boundary W_k consists of linear surfaces that have depressions [8]. The constructed model describes a non-convex and continuous linear programming problem. The definition domain W_k includes all the optimal solutions. Here, we can, at least theoretically, use the program for searching the global extremum for non-linear programing problems and obtain the optimal solution to solve the above problem.

Since minimum and maximum operations [3] are used to construct the Φ-function, problem (12) with constraints (2)–(7) and (9)–(11) belongs to the nonsmooth optimization [8]. According to its construction method, the feasible solution domain W_k can be presented in the form of a subdomain set h (h is a certain number that depends on the number and form of the object) as follows:

$$W_k = \bigcup_{s=1}^{h} W_{ks}, \tag{13}$$

where W_{ks} is described by a system of inequalities with smooth functions in the left-hand side.

The representation of the feasible solution domain in the form of subdomain set (13) allows us to reduce the local extremum of problem (12) with constraints (2)–(7) and (9)–(11) to solving the sequence of nonlinear programming problems using the following algorithm.

Algorithm 1

Step 1. Let us label the starting point $G^s = ((\Delta t^1, z_c^1, x_c^1, y_c^1, \theta_c^1, \theta_l^1, \theta_r^1)^s, \ldots, (\Delta t^i, z_c^i, x_c^i, y_c^i, \theta_c^i, \theta_l^i, \theta_r^i)^s, \ldots, (\Delta t^n, z_c^n, x_c^n, y_c^n, \theta_c^n, \theta_l^n, \theta_r^n)^s)$, $s = 1$, for problem (12) with constraints (2)–(7) and (9)–(11) (it belongs to W_{ks} by its structure).

Step 2. Using the coordinates of the starting point G^s, let us generate a subdomain from the domain W_{ks} (13) that includes this point. If all the domains of W_{ks} are already studied, the solution process ends.

Step 3. Starting from the point G^s, let us find the local minimum of the function $F(G^s)$ in the domain W_{ks}. Let us denote the obtained local extremum point by G^{s+1}.

Step 4. Let us consider that $s = s+1$, and let us transition to Step 2.

Practical research showed that to solve the problem under study, limiting ourselves to two or three Algorithm 1 iterations suffices.

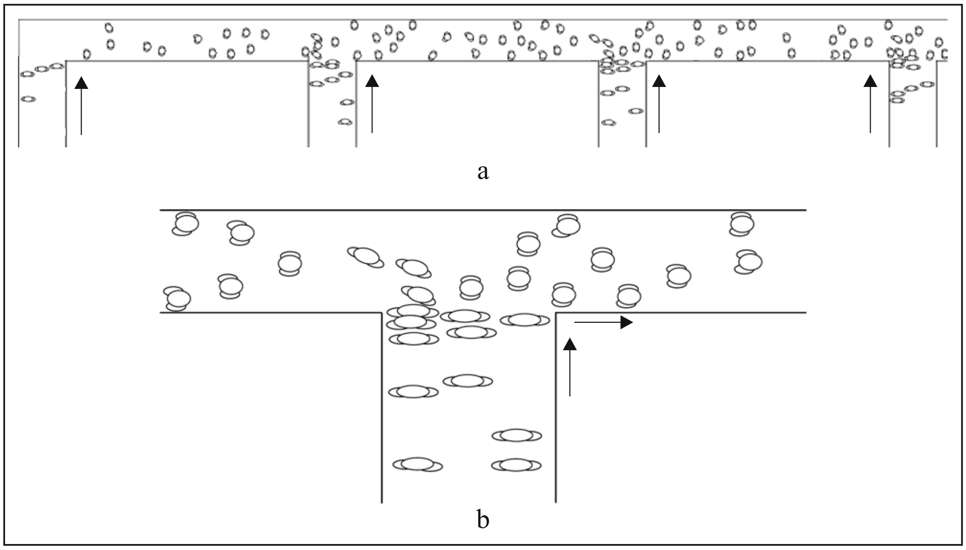

Fig. 4. Configuration of the human placement within 30 sec of their movement where (a) is a configuration fragment of human placement when exciting the third corridor into the main one (b).

Problem decomposition means are proposed that allow us to significantly reduce resource consumption of the optimization process and to use the proposed approach to model a wide spectrum of situations.

This article also developed an approximate problem-solving algorithm that is presented in the form of sequence of the following steps.

Algorithm 2

Step 1. The domain is given in the form of a tree (graph) where edges are corridor segments and vertices are intersections and segment adhesion points. A segment can have a variable width (that changes linearly). A distance to the exit and predominant movement direction are calculated for each segment point.

Step 2. A grid with a small enough step is placed upon the evacuation domain to determine flow density.

Step 3. Complex objects are sorted in the ascending order of the distance to the exit.

Step 4. The local flow density and the predominant movement direction are determined by the sorting fact for each object according to the placement coordinates of the center and the rotation angle of the main ellipse.

Step 5. A finite direction set is considered for the determined predominant movement direction within the limits of the maneuverability angle where there is a direction, along which one can make the maximum movement in a span of a second without violating segment boundaries and fulfilling the conditions of non-intersection with other ellipses. (The movement velocity depends on the local flow density.)

The corresponding software was developed using the algorithms above. The “Evacuation” program that is designated to research the human evacuation model and is based on the emulation of individual human movement [2] was developed. The problem of modeling human movement out of four corridors with identical metric characteristics and with the same number of people inside them (test example from [1]) is considered where the people form local flows that merge in the main corridor leading to the exit. The article considers the influence people exude on one another by force, which reduces movement time in an emergency. Figure 4 presents the configuration of human placemen within 30 sec and a configuration fragment that demonstrates people exiting from one of the local corridors (the third corridor) into the main one.

The human movement modeling problem forms the configuration of their placement in the network of their movement for each fixed movement time.

The labor expenditure of the developed algorithms is estimated. Here, Algorithm 1 is a non-linear algorithm [8] searching for the local extremum of the problem, while Algorithm 2 is an approximate algorithm of the singularly subsequent object movement with the linear labor expenditure [9].

Thus, when modeling the flow movement of humans whose number does not exceed 150, both algorithms are efficient. Under a higher problem dimensionality, the singularly subsequent movement algorithm has to be used.

CONCLUSIONS

The problem of human flow movement belongs to the continuous problem class. When modeling movement, time discreditation is introduced that allows us to reduce the solution to the initial problem to solving a continuous problem sequence at each discrete time interval. Generalized changes for the continuous movement modeling problem are formed at each kth discrete time interval. The representation of generalized changes using the object non-intersection conditions and their placement on the horizontal plane, as well as technological restrictions in accordance with the quality criterion at each fixed moment in time forms their placement configuration. The form of geometric objects that are a combination of three ellipses is considered during active movement. The main ellipse rotates within the maneuverability angle, while the other ellipses rotate in relation to the object adhesion points within the limits of anthropologically feasible angles. The article proposes algorithms for modeling heterogenous human flows that are based on the methods of local optimization of object movement taking into account different spatial forms and metric characteristics. The algorithms are based on the analytical description of conditions of non-intersection of objects and their placement on horizontal paths taking into account continuous translations and rotations of the objects.

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